

Review of Business and Economics

Formerly known as *Tijdschrift voor Economie en Management*

Published quarterly

Volume LIII ▪ number 2 ▪ April-June 2008

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Editor-in-Chief:

C. Crombez
Naamsestraat 69
3000 Leuven

ISSN 0772-7674

PB BC 30664

P808087

Afgiftekantoor: LEUVEN MAIL

Cover design:

Danny Juchtman

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Aggregating and Decomposing Productivity

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ABSTRACT

We illustrate how the aggregate level of total factor productivity, as obtained from aggregate input and output statistics, can be replicated by summing appropriately weighted firm-level measures. This exact aggregation has a number of advantages over existing practices. First, the contribution of different sub-samples to a well-defined aggregate is easily identified. Second, the importance of patterns at the micro level, such as larger size or higher capital intensity for more productive firms, for the aggregate can be calculated. Third, it allows the exact decomposition of aggregate output growth into the contribution of several factors, among which firm-level total factor productivity growth, an inherently relative concept. A sample of all Chinese manufacturing firms with annual sales above five million RMB is used to illustrate the usefulness of the different decompositions.

JEL CODES

O4, D24, L60, J24

KEYWORDS

labor productivity, total factor productivity, exact aggregation, China, manufacturing

RESEARCH FUNDING

Funding was provided by the Connaught Foundation

I. Introduction

Productivity is one of those rare concepts that both micro and macroeconomists use extensively. Trends over time and the impact of capital intensity feature prominently at the macro level, while heterogeneity across firms receives a lot of attention at the micro level. Since the development of large scale firm or plant-level panel data sets in many countries, several studies have bridged the micro-macro gap – see Bartelsman and Dhrymes (1998) for a survey.

In this paper, we focus on exact aggregations and decompositions, which we take to mean the reproduction of aggregate concepts by summing up micro-level measures. A benefit of this approach is that it helps to obtain a better understanding of macro phenomena that have been discovered elsewhere. It frequently occurs in economics that a puzzle at the aggregate level can readily be explained as the result of aggregation over heterogeneous agents.

Consider the following two examples that illustrate the importance of heterogeneity in productivity. In international economics, it seems to be a violation of the theory that firms from one country simultaneously use exports and foreign direct investment to reach foreign customers even in the same industry and country. Helpman, Melitz, and Yeaple (2004) illustrate how this phenomenon occurs naturally if firms are heterogeneous, with more productive firms investing overseas and less productive ones exporting.

In the financial economics literature, several researchers have documented that investors seem to irrationally prefer fund managers that performed well in the previous period, even though all evidence points to below average subsequent returns. Chen, Hong, Huang, and Kubik (2004) illustrate how that pattern can be explained by heterogeneity in the stock-picking abilities of fund managers, which is subject to decreasing returns. Good performance signals skill, but the inflow of new money spreads this asset more thinly.

In both examples, rational actions at the micro level lead to seemingly irrational or illogical macro phenomena. If the agents' actions – trade or investment flows in these examples – are aggregated exactly it would be straightforward to investigate what fraction of the pattern is due to heterogeneity.

It is by now widely accepted that a thorough understanding of aggregate productivity growth has to go beyond the representative agent model (Baily, Hulten, and Campbell 1992). Not only do individual firms enjoy different rates of growth, but the reallocation of input factors between firms of differing productivity level also affects the aggregate. Changes within firms, competition for resources, and changes at the extensive margin (entry and exit) all play a role in the evolution of economic activity. We are obviously not the first to decompose these contributing factors, but earlier work often made aggregations more ad-hoc, losing some of the tight connection.²

The advantage of exactly reproducing the aggregate productivity level that macroeconomists work with lies primarily in the possibility of separating the contribution of different sub-samples. Existing methods do not possess this feature for total factor productivity, which involves a nonlinear calculation. Using our approach, all calculations hold identically at the plant-level, firm-level, or industry-level, and an appropriately weighted sample of firms will reproduce the aggregate productivity level. Averaging the productivity levels of different types of firms,

e.g., private and public firms, also reproduces the aggregate exactly, even for total factor productivity. It further allows for counterfactuals: How much would aggregate output increase if after privatization state-owned firms were to achieve the same productivity level as private firms?

We further contribute to the literature by showing how economy-wide output growth can be decomposed into contributions of several underlying factors. We distinguish the effects of labor input growth and labor productivity growth, and further break down the latter into changes for survivors, entry and exit. Labor productivity growth for survivors is the sum of changes between firms and within firms, and the latter effect is composed of capital deepening and total factor productivity growth. Even though productivity is inherently a relative concept, our approach converts all changes into directly comparable units, output gained or lost. The exact aggregation allows the decomposition to be performed separately for individual sectors or for a particular type of firms.

We do not use ‘exact’ in the sense of exact index numbers (Diewert 1976). The focus in that literature is on obtaining exact micro-level measures of productivity change over time or differences between firms, without estimating the input trade-off that the technology allows. Petrin and Levinsohn (2006) bridge the gap between micro and macro researchers working in that tradition.

We also do not elaborate on the measurement of productivity, which is covered extensively elsewhere (Van Biesebroeck 2007, 2008).³ For any aggregation of multi-factor productivity to work, the input aggregate of all observations has to be comparable, which necessitates the same input coefficients on labor and capital. Törnqvist-type index numbers with observation-specific cost shares will not work, but parametric approaches to productivity measurement will. Throughout, we simply use the average labor share in the economy as weight for labor and enforce constant returns to scale.⁴

The remainder of the paper is organized as follows. First, we discuss how the aggregate productivity level can be reconstructed from micro data and we derive two decompositions. Next, we illustrate how a similar aggregation for productivity growth requires additional terms, naturally leading to the distinction of within and between firm effects. These results are used in Section IV to derive a grand decomposition for aggregate output growth. Finally, in Section V, we use data on Chinese manufacturing firms to illustrate how the different decompositions can be used and the type of insights they can generate. Lessons from the analysis are summarized in Section VI.

II. Aggregate productivity level

A. Labor productivity

Before turning to the decompositions, where our real interest lies, we first discuss how productivity aggregates can be obtained by summing over micro-level measures. Such exact aggregation brings several advantages, which are illustrated explicitly after the discussion of appropriate weights for labor and total factor productivity.

For labor productivity, it is only natural to use labor weights:

$$LP_t = \frac{\sum_{i=1}^N Q_{it}}{\sum_{i=1}^N L_{it}} = \sum_{i=1}^N \left(\frac{L_{it}}{\sum_i L_{it}} \right) \frac{Q_{it}}{L_{it}} = \sum_{i=1}^N \theta_{it}^L LP_{it}.$$

Throughout, we index firms or plants active at time t by $i = 1, \dots, N$.⁵ The output measure we use is value added, netting intermediate inputs from gross output.

Aggregate productivity calculated in this way has the following interpretation:

$$Q_t = \sum_i L_{it} LP_t \tag{1}$$

Aggregate output can be reproduced by employing each worker in the economy at the average labor productivity level. This feature gives labor productivity an absolute interpretation, going beyond its usefulness as a relative concept.

Using output shares to aggregate firms' labor productivity would not be nonsensical, it would just produce a different aggregate with a different meaning. Because the natural interpretation above would not apply anymore, it would become necessary to make the benchmark explicit, as the aggregate would now only be a relative concept. We can show that (Proof is in the Appendix):

Proposition 1. *Aggregating firm-level labor productivity with output (numerator) shares as weights results in a larger aggregate than using labor (denominator) shares.*

$$\sum_i \theta_i^O LP_i \geq \sum_i \theta_i^L LP_i$$

B. Total factor productivity

Aggregate total factor productivity, based on macro statistics, is calculated as

$$TFP_t = \frac{\sum_i Q_{it}}{\left(\sum_i L_{it}\right)^\alpha \left(\sum_i K_{it}\right)^{1-\alpha}},$$

still using value added as output measure. Griliches and Ringstad (1971) illustrate several advantages over gross output based productivity measures. In particular, it results in more comparable measures if firms differ in their use of intermediates or level of vertical integration.

The aggregate is not easily reproduced from firm-level productivity estimates. To construct a weighted average, many papers use output shares (Griliches and Regev 1995; Olley and Pakes 1996), but some advocate the use of aggregate input shares, $\theta_{it}^Z = Z_{it} / \sum_i Z_{it}$ with $Z_{it} = L_{it}^\alpha K_{it}^{1-\alpha}$, which is more similar to the labor shares used for labor productivity (Bartelsman and Dhrymes, 1998). Either choice will not reproduce the aggregate. In the latter case,

$$TFP_t^Z = \sum_i \theta_{it}^Z TFP_{it} = \frac{\sum_i Q_{it}}{\sum_i (L_{it}^\alpha K_{it}^{1-\alpha})},$$

where relative to the aggregate TFP_t concept, the order of summation and geometric weighting in the denominator has been reversed. It still holds, as was the case for labor productivity, that

$$\sum_i \theta_{it}^Q TFP_{it} \geq \sum_i \theta_{it}^Z TFP_{it}.$$

Using numerator weights will produce a larger aggregate than using denominator weights. Given that total factor productivity is inherently a relative concept, this ordering is not very informative in its own right. However, the choice of weights will influence the relative importance of different terms in any decomposition, which makes it particularly attractive to use weights that reconstruct the most frequently used productivity aggregate TFP_t .

To exactly reproduce the aggregate productivity level, the solution is to use

$$\theta_{it}^C = \frac{L_{it}^\alpha K_{it}^{1-\alpha}}{\left(\sum_i L_{it}\right)^\alpha \left(\sum_i K_{it}\right)^{1-\alpha}} = (\theta_{it}^L)^\alpha (\theta_{it}^K)^{1-\alpha}$$

as weight, such that

$$TFP_t = \sum_i \left(\frac{L_{it}}{\sum_i L_{it}} \right)^\alpha \left(\frac{K_{it}}{\sum_i K_{it}} \right)^{1-\alpha} \frac{Q_{it}}{L_{it}^\alpha K_{it}^{1-\alpha}} = \sum_i \theta_{it}^C TFP_{it},$$

as desired. As far as we are aware, these weights have not been used before in the literature. An issue is that they do not sum to unity.

Proposition 2. *The weights that reproduce aggregate total factor productivity as the sum of firm-level productivity sum to less than one.*

$$\sum_i \theta_{it}^C \leq 1$$

(Proof is in the Appendix.)

The sum of weights reaches a minimum for $\alpha = 0.5$, is decreasing in the correlation between L and K , and approaches one if the correlation between the two inputs goes to zero.

C. Advantages of exact aggregation

Being able to exactly reproduce aggregate statistics has a number of advantages. We identify at least four.

First, the calculations hold at different levels of aggregation. It holds that (k is used to index industries, i for plants or firms)

$$LP_t = \sum_{k=1}^K \sum_{i=1}^{N_k} \frac{L_{kit}}{\sum_k \sum_i L_{kit}} LP_{kit} = \sum_{k=1}^K \frac{L_{kt}}{\sum_k L_{kt}} \underbrace{\sum_{i=1}^{N_k} \frac{L_{kit}}{L_{kt}} LP_{kit}}_k = \sum_{i=1}^K \theta_{kt}^L LP_{kt},$$

with $L_{kt} = \sum_{i \in k} L_{kit}$. Aggregate productivity is the labor share weighted sum of micro-level productivity, at any level of disaggregation. The same holds for total factor productivity if the composite input weights (C weights) are used.

Second, if one works with a survey of firms, as opposed to the full census, one can straightforwardly adjust the weights to reflect the sampling frame, see for example Griliches and Regev (1995). Similarly, Van Biesebroeck (2003) adjusts the labor share of each firm with the probability that it employs the ‘lean’ or ‘mass’ technology, to separate the contributions of either technology. As the probabilities sum to one, it still reproduces the aggregate productivity level.

Third, calculating subtotals is not limited to industries. One could just as easily separate the contribution of different types of firms, e.g., exporters and non-exporters (Van Biesebroeck 2005a), domestic and foreign-owned entities (Brandt, Van Biesebroeck, Wang, and Zhang 2007), or new and old firms (Van Biesebroeck 2002), and obtain the average productivity for each group. The weighted average of the subtotals will again reproduce the aggregate.

Fourth, the fact that the input-weighted sum of average productivity reproduces total output, can be used to conduct counterfactual experiments. For example, if privately-owned firms (*PR*) are found to operate at a higher level of productivity, we can estimate the potential output gain in the absence of any re-allocation of factor inputs if all state-owned firms (*ST*) were to be privatized. In the case of labor productivity, we can use equation (1) to calculate that

$$\Delta Q_t = \sum_{j \in ST} L_{jt} (LP_t^{PR} - LP_t^{ST})$$

A similar counterfactual is possible for total factor productivity, only we have to use the following formula to construct aggregate output

$$\sum_{i=1}^N Q_{it} = \left(\sum_i L_{it} TFP_t \right)^\alpha \left(\sum_i K_{it} TFP_t \right)^{1-\alpha},$$

which will not eliminate the (unchanged) contribution of the already private firms from the ΔQ_t calculation. Moreover, we have to keep in mind that the θ^C shares will not sum to unity when we substitute $\theta_{PR}^C * TFP_t^{PR} + \theta_{ST}^C * TFP_t^{ST}$ for TFP_t . However, we can rewrite the change in output as

$$\begin{aligned} \Delta Q_t &= \sum_{j \in ST} \frac{L_{jt}}{\left(L_t^{ST} / K_t^{ST} \right)^{1-\alpha}} \left(TFP_t^{PR} - TFP_t^{ST} \right) \text{ or} \\ &= \sum_{j \in ST} \frac{K_{jt}}{\left(K_t^{ST} / L_t^{ST} \right)^\alpha} \left(TFP_t^{PR} - TFP_t^{ST} \right), \end{aligned}$$

which adjusts the inputs of the affected firms for K_t^{ST} / L_t^{ST} , the average capital-labor ratio for the affected firms.⁶

D. Decomposing the productivity level

We consider two different decompositions. First, we study the link between size and firm-level productivity as a determinant of the aggregate productivity level, which is equally relevant for labor and total factor productivity. Second, we investigate to what extent aggregate labor productivity is the result of a positive link between total factor productivity and capital intensity.

As aggregate productivity can be defined as the share weighted sum of firm-level productivity, it is immediate that the aggregate can be high because average productivity levels are high or because the most productive firms employ most resources. Olley and Pakes (1996) perform a decomposition for total factor productivity into an unweighted average and a term representing the correlation between productivity and market share (between productivity and resource use with our

weights). Representing productivity by P , denoting either labor or total factor productivity, we can write

$$\begin{aligned}
 P_t &= \frac{1}{N} \sum_i P_{it} + \sum_i \left(\theta_{it} - \frac{1}{N} \right) P_{it} \\
 &= \bar{P}_t + \sum_i \Delta \theta_{it} \Delta P_{it}.
 \end{aligned}
 \tag{2}$$

Replacing the multiplication by P_{it} in the second term of the first line by $\Delta P_{it} = (P_{it} - P_t)$ or $\Delta P_{it}^* = (P_{it} - \bar{P}_t)$ is only possible if the shares sum to unity, i.e., not for θ^C (Proposition 2). For total factor productivity a normalization is needed and dividing every term by P_0 is an intuitive solution.

This decomposition can be used to compare how tightly resource use and productivity are associated in different sectors or for different types of firms by limiting the summations to a subset of observations. It can also be used to verify whether the correlation between resource use and productivity is becoming stronger over time.

The choice of weights is neither obvious, nor innocuous. Olley and Pakes (1996) used output weights, while Bartelsman and Dhrymes (1998) used aggregate input share weights (Z weights) for their identical graphical decomposition. We can show that.

Corollary 3. *Using output (numerator) weights amplifies the relative importance of the correlation term.*

For example, in the case of labor productivity, it follows directly from Proposition 1 and the above decomposition that

$$\sum_i \Delta \theta_{it}^L \Delta LP_{it} \leq \sum_i \Delta \theta_{it}^Q \Delta LP_{it},$$

while the unweighted average, the first term, is obviously unaffected by the choice of weights. Defining the aggregate using output weights intensifies, by construction, the association between productivity and a firm’s relative share.

We can write firm-level labor productivity as the product of two terms, $TFP_{it} * (K_{it} / L_{it})^{1-\alpha}$, total factor productivity, which captures relative efficiency or superior technology, and capital intensity.⁷ Two alternative decompositions can be used to investigate the relative importance of either factor as follows:

$$LP_t = \sum_i \theta_{it}^L TFP_{it} \left(\frac{K_t}{L_t} \right)^{1-\alpha} + \sum_i \theta_{it}^L TFP_{it} \left[\left(\frac{K_{it}}{L_{it}} \right)^{1-\alpha} - \left(\frac{K_t}{L_t} \right)^{1-\alpha} \right]
 \tag{3}$$

$$= \sum_i \theta_{it}^L TFP_t \left(\frac{K_{it}}{L_{it}} \right)^{1-\alpha} + \sum_i \theta_{it}^L (TFP_{it} - TFP_t) \left(\frac{K_{it}}{L_{it}} \right)^{1-\alpha}, \quad (4)$$

where $K_t / L_t = \sum_i K_{it} / \sum_i L_{it}$ and $TFP_t = \sum_i \theta_{it}^C TFP_{it}$.

The first terms calculate average labor productivity holding different things constant. In equation (3), each firm's efficiency level is weighed by its employment share and the average capital intensity in the economy. This is what aggregate labor productivity would be if the employment size and efficiency level of each firm were held constant and the capital stock distributed evenly such that each employee had the same capital to work with. Because of decreasing return to capital, redistributing the capital stock in this way without changing productivity is likely to lead to higher output per worker on average. We expect the first term to exceed LP_t .

In equation (4), total factor productivity is held constant when calculating average labor productivity in the first term. This is what aggregate labor productivity would be if all firms in the economy were equally efficient, irrespective of their employment share and capital intensity. The first term can be rewritten as $LP_t \sum_i (\theta_{it}^L)^\alpha (\theta_{it}^K)^{1-\alpha}$, which according to Proposition 2 is weakly lower than LP_t . If all firms had the same capital-labor ratio, the first term would replicate LP_t as it would make labor shares uncorrelated with capital intensity. If firms that use a lot of capital failed to improve their efficiency level above the average, the aggregate labor productivity level would suffer and this would show up as a low (but still positive) value for the second term.

In each version, (3) or (4), the second term captures whether highly efficient firms – those with high TFP_{it} – also operate with a high capital stock per worker. It approximates the correlation between efficiency and capital intensity, just as the previous decomposition in (2) identified the correlation between productivity and size. It follows from the above discussion of the first terms that it is the absolute value of the second term that matters, with the expectation that its sign will be negative in (3) and positive, by construction, in (4). We illustrate the different decompositions in Section V.

III. Productivity growth

A. Entry and exit, within and between

Productivity measurement is inherently a relative exercise, only defined with respect to a specific production function (Van Biesebroeck 2007). As units of capital

and numbers of workers are aggregated, a normalization is needed to make it a unit-free concept. The most straightforward comparison is between productivity in two time periods. In the following, productivity differences are expressed relative to the initial productivity level, making all formulas equally applicable to labor and to total factor productivity. Aggregate productivity growth is an important concept. Basu and Fernald (2002) show that total productivity growth is proportional to aggregate change in welfare under mild conditions.⁸

We have already shown that properly weighted average firm-level productivity reproduces the aggregate productivity level. For productivity growth, there are two complications.⁹ In different time periods, different sets of firms will be active, such that the summation is over a different set. The contribution of entering and exiting firms has to be considered explicitly. Moreover, the weight of each firm will generally differ in the two years that are compared and such changes in weight will have an effect on aggregate productivity growth depending on whether growing firms had below or above average productivity. The weighted average of firm-level productivity growth will be only one of several contributing factors to the aggregate growth.

Straightforward algebra leads to a decomposition of aggregate productivity growth, $PG_t = (P_t - P_{t-1}) / P_{t-1}$, as

$$\begin{aligned}
 PG_t &= \left(\sum_{l=i,n} \theta_{lt} P_{lt} - \sum_{k=i,x} \theta_{kt-1} P_{kt-1} \right) / P_{t-1} & (5) \\
 &= \underbrace{\sum_i^{\text{survivor}} \frac{(\theta_{it} P_{it} - \theta_{it-1} P_{it-1})}{P_{t-1}}}_{(S)} + \sum_n^{\text{enter}} \theta_{nt} \frac{P_{nt}}{P_{t-1}} - \sum_x^{\text{exit}} \theta_{xt-1} \frac{P_{xt-1}}{P_{t-1}},
 \end{aligned}$$

which separates the contribution from surviving firms (indexed by i and denoted by (S)) in the first term from that of entering or exiting firms (indexed by n and x , respectively). Because we are again replicating the aggregate, we can further decompose each of the summations into the contribution of different types of firms, e.g., by size class (Van Biesebroeck 2005b).

In this formulation, firms that newly entered in period t make a positive contribution to aggregate productivity growth whether their productivity level is above or below the initial aggregate productivity. Similarly, an exiting firm will negatively affect the aggregate as its productivity level goes from P_{xt-1} to zero at time t . Given that our objective will be to trace where additions to aggregate output originate, in the next section, this does not seem unreasonable. Each firm's contribution is increasing in its relative productivity level.

An alternative is to express all micro-level productivity terms in (5) as deviations from the initial aggregate, i.e., subtracting P_{t-1} . For labor productivity this would

not require any additional changes, but for total factor productivity it would introduce the additional term $(\sum_{k \in i, n} \theta_{kt}^C - \sum_{l \in i, x} \theta_{lt-1}^C) TFP_{t-1}$ as the weights do not sum to unity. In deviation form, entering or exiting firms only contribute to aggregate productivity growth if their productivity level differs from the initial aggregate productivity.

Even if newly entering firms have below average productivity at first, which is often the case, net entry can increase aggregate productivity if exiting firms were even less productive, which is also a common phenomenon, dubbed the “shadow of death” by Griliches and Regev (1995). Moreover, much evidence points to a high exit rate for new firms in the first years, with above average productivity growth rates for surviving entrants (Bartelsman and Doms, 2000). As a result, the importance of net entry to aggregate productivity growth tends to be increasing in the time elapsed between t and $t - 1$.

The aggregate productivity growth for the balanced panel, ignoring entry and exit, as captured by the first term in equation (5), can be decomposed further as

$$(S) = \sum_i (\theta_{it-1} PG_{it} + \Delta\theta_{it} + \Delta\theta_{it} PG_{it}) \frac{P_{it-1}}{P_{t-1}},$$

where $PG_{it} = (P_{it} - P_{it-1}) / P_{it-1}$. The first term is the within-firm productivity growth contribution, averaging firm-level growth weighed by the initial share. The second term is the between-firm contribution, summing firm-level share changes. It is more likely to be positive if incumbents increase their overall share, and especially if the additions are concentrated in more productive firms. The third term is a covariance term, which will make a positive contribution if firms that increased productivity were more likely to do this by generating additional output rather than by decreasing their input use. In each term, observations are weighted by their relative productivity, normalized by the average initial productivity level.

Without changing the relative weight of the different terms, we can work out the multiplication and express the decomposition as

$$(S) = \sum_i \left(\theta_{it-1} \frac{\Delta P_{it}}{P_{t-1}} + \Delta\theta_{it} \frac{P_{it-1}}{P_{t-1}} + \Delta\theta_{it} \frac{\Delta P_{it}}{P_{t-1}} \right). \quad (6)$$

Yet an alternative is to split the covariance term equally between the first two terms by using average instead of initial weights, as was done by Griliches and Regev (1995). This results in

$$(S) = \sum_i \left(\bar{\theta}_i \frac{\Delta P_{it}}{P_{t-1}} + \Delta\theta_{it} \frac{\bar{P}_i}{P_{t-1}} \right),$$

where $\bar{\theta}_i$ is defined as $(\theta_{it} + \theta_{it-1}) / 2$ and similarly for \bar{P}_i .¹⁰ In this case, it is perhaps more intuitive to use $\bar{P} = (P_t + P_{t-1}) / 2$ to normalize the firm-level productivity levels that multiply the share difference and the entry and exit terms.

B. Comparison with the literature

In practice, many researchers have aggregated firm-level productivity levels geometrically, which facilitates the linear decomposition of aggregate productivity growth in within and between-firm effects. Baily, *et al.* (1992) pioneered these decompositions, defining the aggregate productivity level as $P_t = \sum_i \theta_{it}^Q \ln P_{it}$ and productivity growth as $\Delta \ln P_t = \ln P_t - \ln P_{t-1}$. Haltiwanger (1997) improved their decomposition to better account for unbalanced panels and wrote

$$\Delta \ln P_t = \sum_i^{\text{survivor}} \left(\theta_{it-1}^Q \Delta \ln P_{it} + \Delta \theta_{it}^Q (\ln P_{it-1} - \ln P_{t-1}) + \Delta \theta_{it}^Q \Delta \ln P_{it} \right) \quad (7)$$

$$+ \sum_{nt}^{\text{enter}} \theta_{nt}^Q (\ln P_{nt} - \ln P_{t-1}) - \sum_x^{\text{exit}} \theta_{xt-1}^Q (\ln P_{xt-1} - \ln P_{t-1}).$$

As before, the normalization of firm-level productivity level by $\ln P_{t-1}$ relies on the shares of the second, fourth and fifth term summing to zero, which works for the output weights that Haltiwanger (1997) uses. Van Biesebroeck (2005b) employs this decomposition to look specifically at the reallocation of workers across firms, using labor shares both for labor productivity and for total factor productivity growth.

As Fox (2003) illustrates, the between and covariance terms in the above decompositions, both (6) and (7), can give rise to counterintuitive results. For example, it is possible that all firms increase their productivity, but aggregate productivity falls. It is similarly possible that sector-by-sector, one country has consistently higher productivity growth than another, but lower aggregate productivity growth. The reason is that the shares are not held constant in the comparison between P_t and P_{t-1} . While this is an unattractive property for a quantity index, it is an inherent feature of our measure of aggregate productivity which has a well defined definition in its own right, as discussed in the previous section.

Petrin and Levinsohn (2006) illustrate further that the between and covariance terms do not contribute to aggregate welfare change if markets are perfectly competitive. If the marginal productivity of workers is equalized across firms, moving workers from firms with low to high average productivity does not contribute to aggregate welfare. Only if markets are imperfectly competitive, and factors are not rewarded at their marginal productivity, does the between component play a role in welfare. As such, the interpretation of the between term is theoretically ambiguous.

Abstracting from entry and exit and market imperfections, the properly weighted average firm-level productivity growth by itself replicates aggregate technical change.

With the explicit objective to study welfare change, Petrin and Levinsohn (2006) start from the Törnqvist-Divisia quantity index as a discrete approximation of the aggregate productivity growth, in our notation

$$d\widehat{TFP}_t = \sum_i^{\text{survivor}} \bar{\theta}_i^{O'} \Delta \ln TFP_{it} + \sum_n^{\text{enter}} \theta_{nt}^{O'} / 2 \hat{\Delta} \ln TFP_{nt} + \sum_x^{\text{exit}} \theta_{xt-1}^{O'} / 2 \hat{\Delta} \ln TFP_{xt}.$$

To calculate the contribution of entrants (exiting firms), an approximation is necessary as their exact productivity level at the time of entry (exit) is unobservable. Regression analysis can be used to construct the second term in $\hat{\Delta} \ln TFP_{nt} = \ln TFP_{nt} - \ln \widehat{TFP}_{nt}$ and the first term in $\hat{\Delta} \ln TFP_{xt} = \ln \widehat{TFP}_{xt} - \ln TFP_{nt-1}$, where τ is the time of entry or exit. The proposed weights for entrants and exiting firms are upper bounds on the theoretically correct weights.

An additional difference in the Petrin-Levinsohn decomposition is the use of ‘Domar’ weights, which are the correct ones if the aim is to approximate welfare gains associated with productivity growth. If productivity is calculated from a gross output production function, the correct weights equal $P_i(Q_i + M_i) / \sum_i P_i Q_i = \theta_i^O(Q_i + M_i) / Q_i$, scaling up value added weights by the ratio of gross output to value added.¹¹ For the value added production function that we have worked with, these boil down to average value added shares. Recall that the composite input weights (C weights) replicate the change in aggregate productivity, but only if the between and covariance terms are included.

IV. Output growth

Fox (2003) criticizes the definition of aggregate productivity growth above, because it mixes ‘price’ and ‘quantity’ changes, making it a value concept, not a quantity index.¹² However, a major motivation for us to decompose productivity is to understand the sources of output growth, which is inherently a value concept. Output has a meaningful interpretation in its own right, at an absolute level, not only in relative terms as an index. The same is true for aggregate labor productivity growth, which indicates how much additional output is produced by the average worker.

At the aggregate level, it is obvious that

$$\ln \frac{Q}{Q_{t-1}} = \ln \frac{Q_t / L_t}{Q_{t-1} / L_{t-1}} + \ln \frac{L_t}{L_{t-1}}.$$

Output growth is the sum of labor productivity growth and labor input growth. As discussed earlier, labor productivity growth comes about by more efficient production – improvements *within* firms – or by better allocation of inputs – workers moving *between* firms.

In order to replicate aggregate output change, we cannot define the aggregate as the geometric average of firm-level productivity, as is common in the literature, because no weighting scheme will ever equalize $\ln Q_t / L_t$ to $\sum_i \theta_{it} \ln LP_{it}$. To build up aggregate output change from micro-level changes, we start with

$$\Delta Q_t = \Delta LP_t L_{t-1} + LP_{t-1} \Delta L_t + \Delta LP_t \Delta L_t.$$

The second term, the mobilization of new resources, is inherently an aggregate change. As the product of two percentage changes, the third term is only second order. Substituting equations (5) and (6), the first term can be decomposed further as

$$\begin{aligned} \Delta LP_t L_{t-1} = & \sum_i^{survivor} \left(L_{it-1} \Delta LP_{it} + \tilde{\Delta} L_{it} LP_{it-1} + L_{t-1} \Delta \theta_{it}^L \Delta LP_{it} \right) \\ & + \sum_n^{enter} L_{nt} / (1 + \Delta L_t / L_t) LP_{nt} - \sum_x^{exit} L_{xt-1} LP_{xt-1}, \end{aligned}$$

with the modified difference $\tilde{\Delta} L_{it}$ defined as $L_{it} / (1 + \Delta L_t / L_t) - LP_{it-1}$, to control for the growth in the total labor force. The same correction is applied to employment of the entering firms. As the equation is decomposing changes in the aggregate output level, we have to use employment levels rather than shares as weight. The first term, which captures the within-firm change, can be further decomposed, recognizing again that labor productivity is the product of total factor productivity growth (TFP_{it}) and capital intensity (K_{it} / L_{it})^{1- α} . After some algebra, it can be written as

$$\sum_i L_{it-1} \Delta LP_{it} = \sum_i \left[Z_{it-1} \Delta TFP_{it} + L_{it-1} TFP_{it-1} \Delta (K_{it} / L_{it})^{1-\alpha} + L_{it-1} \Delta TFP_{it} \Delta (K_{it} / L_{it})^{1-\alpha} \right]$$

where $\Delta (K_{it} / L_{it})^{1-\alpha} = (K_{it} / L_{it})^{1-\alpha} - (K_{it-1} / L_{it-1})^{1-\alpha}$. Firm-level labor productivity growth is the result of technological improvements, in the first term, and capital deepening, in the second term. Total factor productivity changes are weighted by the composite input aggregate, similarly as we did for the productivity level. Changes in the firm-level capital-labor ratios are weighted by the firm's initial employment and its total factor productivity level. The third term will make a positive contribution if total factor productivity growth is more likely to be the result of output expansions being accompanied with capital deepening rather than with less

capital-intensive production. Only in the former case are input factors used more effectively.

It is possible to avoid the third, covariance term in each of the three decompositions, by using average weights. Putting everything together, we can factor output change as follows:

$$\begin{aligned}\Delta Q_t &= \bar{L}\Delta LP_t + \overline{LP}\Delta L_t; \\ \Delta LP_t &= \sum_i^{survivor} \left(\bar{\theta}_i^L \Delta LP_{it} + \overline{LP}_i \Delta \theta_{it}^L \right) + \sum_n^{enter} \theta_{nt}^L LP_{nt} - \sum_x^{exit} \theta_{xt-1}^L LP_{xt-1}; \\ \sum_i \bar{\theta}_i^L \Delta LP_{it} &= \sum_i \left[\bar{\theta}_i^L \left(\overline{K_i / L_i} \right)^{1-\alpha} \Delta TFP_{it} + \bar{\theta}_i^L \overline{TFP}_i \Delta \left(K_{it} / L_{it} \right)^{1-\alpha} \right],\end{aligned}\tag{8}$$

with $\left(\overline{K_i / L_i} \right)^{1-\alpha} = \left[\left(K_{it} / L_{it} \right)^{1-\alpha} + \left(K_{it-1} / L_{it-1} \right)^{1-\alpha} \right] / 2$. At each stage, the contribution of the covariance term is divided equally between the first two terms: (i) productivity growth versus input use, (ii) within versus between changes, and (iii) total factor productivity change versus capital deepening. We can express everything in percentage changes by dividing in the first equality by \overline{LLP} and in the other two by LP .¹³

Because the labor shares sum to unity in each period, we can rewrite the second line in decomposition (8) as

$$\begin{aligned}\Delta LP_t &= \sum_i^{survivor} \left(\bar{\theta}_i^L \Delta LP_{it} + \left(\overline{LP}_i - LP_{t-1} \right) \Delta \theta_{it}^L \right) \\ &\quad + \sum_n^{enter} \theta_{nt}^L \left(LP_{nt} - LP_{t-1} \right) - \sum_x^{exit} \theta_{xt-1}^L \left(LP_{xt-1} - LP_{t-1} \right),\end{aligned}\tag{9}$$

similar to the decomposition in Haltiwanger (1997). All productivity levels are now expressed relative to the initial aggregate productivity level and movements of workers only affect aggregate productivity to the extent that the productivity level of expanding and contracting firms differs from the average. As the left-hand side is unchanged, this will only affect the relative importance of the three rightmost terms.

V. Illustration for Chinese manufacturing (2000-2005)

The data used to illustrate the different decompositions is from the Annual Surveys of Industrial Production conducted by the Chinese government's National Bureau of Statistics (NBS). It is a census of all state-owned enterprises and all privately-owned firms with sales exceeding 5 million RMB (approximately \$600,000). We use

an unbalanced panel of firms for the years 2000 and 2005, limited to those active in the manufacturing sector.

Real values for value added and the capital stock are constructed using 4-digit industry deflators for output and intermediate inputs and a separate capital goods deflator. Observations with missing values for value added, employment or capital are omitted. We further drop firms in the top and bottom 0.5 percentile in terms of labor productivity and employment or capital growth, in order to trim the sample of possibly miscoded observations or erroneous firm-matches. Summary statistics are in Table 1. Brandt, *et al.* (2007) provide additional details on the construction of the data set and compare several patterns with those uncovered in the widely used U.S. Longitudinal Research Database.

Table 1. Summary statistics.

	2000		2005		2000-2005 growth	
	mean	st. dev.	mean	st. dev.	mean	st. dev.
number of firms	124,523		232,266		58,346	
employment	317	1,200	236	853	67%	350%
sales/worker	285,779	946,171	340,629	458,069	124%	1518%
capital/worker	74,324	467,015	71,575	160,323	122%	501%
va/worker	64,521	125,936	103,830	139,973	220%	890%
exporter dummy	0.26	0.44	0.29	0.46		
public dummy	0.53	0.50	0.12	0.33		
foreign dummy	0.20	0.40	0.22	0.41		

Note: all values (sales, capital, and value added) are in real terms (2000 RMB).

From the 124,523 firms active in 2000, 47% survive until 2005. They are joined by 173,920 firms that started up between 2000 and 2005. Some of these ‘entrants’ are restructured state-owned enterprises, note that the share of public enterprises falls from 53% to 12%, or small private firms that cross the threshold for inclusion in the sample.¹⁴ Average employment per firm declines from 317 to 236 workers, which makes sense as entering firms tend to be smaller on average. Average employment growth for surviving firms is a robust 67% over the five year period, but standard deviations for all firm-level growth rates are quite large.

From these summary statistics, we can already anticipate large productivity gains. While sales per worker increases on average by 124% for surviving firms, value added per worker increases by 220%. Looking at all active firms, the difference is even starker: the unweighted average sales per worker increases by less than 20%, while the corresponding increase for value added per worker exceeds 61%.

Table 2. Productivity level decompositions.

	Labor productivity		Total factor productivity	
	2000	2005	2000	2005
Index (2000 aggregate = 1)	1.00	2.28	1.00	2.03
(Aggregate, 2000 RMB)	43,986	100,100		
Non-exporters	0.91	0.93	0.97	0.97
Exporters	1.10	1.06	1.03	1.03
Potential output growth (1)	10.4%	6.3%	3.0%	3.0%
Public firms	0.81	0.89	0.87	0.86
Private, domestic firms	1.03	0.94	1.01	0.94
Foreign firms	1.47	1.16	1.30	1.16
Potential output growth (2)	11.3%	0.6%	7.0%	1.0%
(a) Size-productivity decompositions (L weights for LP, C weights for TFP)				
Unweighted average	1.47	1.04	1.98	1.70
Size-productivity covariance	-0.47	-0.04	-0.98	-0.70
(Q weights)				
Unweighted average	0.38	0.34	0.60	0.55
Size-productivity covariance	0.62	0.66	0.40	0.45
(Z weights)				
Unweighted average	0.97	0.72	1.65	1.39
Size-productivity covariance	0.03	0.28	-0.65	-0.39
(b) Technology-capital intensity decompositions				
Constant K/L	1.26	1.37		
TFP – $\Delta K/L$ covariance	-0.26	-0.37		
Constant TFP	0.83	0.82		
ΔTFP – K/L covariance	0.17	0.18		

Notes: “Potential output growth (1)” refers to a hypothetical situation where all non-exporters achieve the same productivity level as exporters; “Potential output growth (2)” to a situation where public enterprises achieve the same productivity level as domestic private firms. The different weights are defined in Section II.

Aggregate labor productivity in 2000 was 43,986 RMB and increased by 128% to 100,100 RMB in 2005, see Table 2. For total factor productivity, the absolute level is not a meaningful statistic, but the index increased 103%. These numbers can be obtained using aggregate input and output statistics or by averaging firm-level productivity measures using appropriate weights.

Exporters are more productive than non-exporters using either measure, but for labor productivity the gap had narrowed somewhat by 2005. Public enterprises are shown to be a lot less and foreign firms a lot more productive than private, domestic firms. Differences with the aggregate are similar for labor and total factor productivity, suggesting that they are not the result of capital intensity differences. The much smaller differences by ownership in 2005 are to a large extent the result of less productive public firms disappearing. In 2005, private domestic firms make up two thirds of the sample, compared to only one quarter initially.

If all non-exporters were to attain the same total factor productivity level as exporting firms, aggregate output would increase by approximately 3%.¹⁵ If the same feat could be accomplished for labor productivity, output would even have increased by 10.4% in 2000 and by 6.3% in 2005, but this would be hard to achieve without reallocating capital. If public enterprises were to attain the total factor productivity level of private enterprises, total value added would have been 7.0% higher in 2000, but only 1.0% higher in 2005, which indicates that most of the inefficiencies associated with the state-owned enterprise sector had already been eliminated by 2005.

The next decompositions confirm the predictions of Corollary 3. The correlation between size and productivity is biased upward if output weights (Q weights) are used and the difference with input weights (L weights for LP, C weights for TFP) is large. Comparing the results for 2000 and 2005, we find that the correlation has increased, irrespective of the type of weight used. While firms with a lot of employees clearly had below average labor productivity in 2000, by 2005 the negative correlation had all but disappeared. The negative association between total resource use (Z weights) and total factor productivity was especially strong in 2000, possibly reflecting the overinvestment in the unproductive state sector, but by 2005 this correlation had weakened too.

Finally, the last two decompositions allow an investigation of the relative importance of technology and capital differences in labor productivity. First, results from the decomposition with “constant K/L”, following equation (3), suggest that the importance of capital has increased over time. If capital could have been reallocated across firms to equalize the capital-labor ratio without affecting total factor productivity levels (and employment shares), aggregate labor productivity would have been 26% higher in 2000 and even 37% higher in 2005. The differences reflect that firms operating with higher capital intensity are not able to increase their total factor productivity level in proportion.

The pattern in the next decomposition with “constant TFP”, following equation (4), qualifies this finding. If firms with high and low capital stock would have attained the same, average total factor productivity level, aggregate labor productivity would have suffered by 17% in 2000. This fraction represents the above average total factor productivity level that capital-intensive firms achieved, even though the increase is not in proportion to their capital advantage. The capital-productivity association did become slightly higher over time, indicated by the increase of the second term to 18% by 2005.

Table 3. Output growth decompositions for the entire manufacturing sector (2000-2005).

	with covariance terms		with average weights		with average weights and normalized productivity levels	
	billion RMB	percentage	billion RMB	percentage	billion RMB	percentage
Growth in:						
1. Aggregate output (1 = 2 + 3 + 4)	3,758	216.3%	3,758	110.5%	3,758	110.5%
2. Covariance 1	864	49.7%	-	-	-	-
3. Labor input	677	39.0%	1,109	32.6%	1,109	32.6%
4. Labor productivity (4 = 5 + 6 + 7)	2,217	127.6%	2,649	77.9%	2,649	77.9%
5. + Entry	2,067	119.0%	2,470	72.6%	1,322	38.9%
6. - Exit	-572	-32.9%	-684	-20.1%	165	4.9%
7. Survivors (7 = 8 + 9 + 10)	722	41.6%	863	25.4%	1,162	34.2%
8. Covariance 2	-509	-29.3%	-	-	-	-
9. Between	50	2.9%	-245	-7.2%	54	1.6%
10. Within (10 = 11 + 12 + 13)	1,181	68.0%	1,108	32.6%	1,108	32.6%
11. Covariance 3	53	3.0%	-	-	-	-
12. Technology	874	50.3%	905	26.6%	905	26.6%
13. Capital intensity	255	14.7%	202	5.9%	202	5.9%

Notes: Rounding introduces slight discrepancies in the totals. In the results that use average weights, percentage numbers are obtained by dividing each RMB amount by (L*LP), which is 6.1% lower than average Q (see footnote 12). On the other hand, average LP is 1.9% higher than (TFP*K/L1-a), which would be the more intuitive denominator for the last two terms.

In Table 3, we decompose aggregate output growth for the total manufacturing sector into the different terms derived in Section 4. The first two columns are for the decompositions with covariance terms and the percentage changes are relative to aggregate output in 2000. The next columns use the alternative, average weights and divide by $\overline{LP\overline{L}}$ to obtain percentages, which approximates average output. In the last two columns, all firm-level productivity levels are normalized by the initial aggregate productivity, as in equation (9), which only makes a difference for the entry, exit, and between terms. Several of the intermediate totals are further decomposed, which is indicated by the bracketed expressions.

Over the 2000-2005 period, 15.4 million workers were added to the manufacturing workforce. While this input contribution is non-negligible, the increase in labor productivity plays an even larger role. Foremost among the contributions to labor productivity is the effect of net entry. More than half of the additional output is produced by firms that did not exist yet in 2000. In contrast, the contribution of surviving firms never exceeds one third of the output gain and in the first decomposition it is even less than one fifth.

Even when we limit the entry effect to only contribute positively when the productivity level of entrants exceeds the initial productivity, in the last two columns, entry accounts for more than a third of the output addition. Even exit makes a positive contribution if productivity levels are normalized this way, indicating that less productive firms are forced to exit the industry. In the first four columns, exit has a negative impact by construction, but the output loss by exiting firms is more than made up by output additions of entrants.

The contribution of firm-level productivity growth, the within term, accounts for slightly less than one third of the output increase. The bulk of it comes from total factor productivity growth ('Technology'), not capital deepening. The between term, which has generated some controversy in the literature, is consistently small.

Finally, in Table 4 we repeat the decomposition with average weights and normalized productivity levels for two subsamples: foreign-owned firms and the textile sector, broadly defined.¹⁶ For convenience, we repeat the total manufacturing results in the first column, but the intermediate totals are omitted. The percentages now indicate the contribution of the different terms in the total output gain and sum to 100%.

Both of these subsectors have boomed in recent years; employment growth in these sectors accounts for almost the entire net manufacturing employment growth. In contrast, the two sectors only account for slightly more than half the aggregate output increase. It is no surprise then, that the input contribution is notably larger than for the economy at large, accounting for respectively 59% and 43% of the output gain. For foreign-owned firms, the contribution of total factor productivity

growth by surviving firms stands out, as it makes up 52% of the labor productivity growth, compared to only 34% for the total economy. For textile firms, the contribution of highly productive entrants stands out.

Table 4. Output growth decompositions for different sub-sectors (2000-2005).

	Entire economy		Foreign-owned firms		Textile/apparel/leather firms	
	billion RMB	percentage contribution	billion RMB	percentage contribution	billion RMB	percentage contribution
	(+ 15.4 million workers) (+ 110.5% output)		(+ 10.0 million workers) (+ 138.0% output)		(+ 4.2 million workers) (+ 105.2% output)	
ΔOutput	3,758		1,535		396	
Labor input	1,109	29.5%	902	58.8%	168	42.5%
+ Entry	1,322	35.2%	348	22.7%	127	32.1%
- Exit	165	4.4%	44	2.8%	11	2.8%
Between	54	1.4%	-78	-5.1%	13	3.4%
Technology	905	24.1%	329	21.4%	62	15.7%
Capital intensity	202	5.4%	-11	-0.7%	13	3.4%

Note: The percentages now represent the contribution of the different terms to the aggregate output gain; they sum to 100%. The decompositions are with average weights and normalized productivity levels, corresponding to the last two columns in Table 3. The following three subtotals are not shown anymore, but can be calculated easily: (i) the labor productivity growth contribution is the sum of the last five terms; (ii) the contribution of survivors is Between + Technology + Capital intensity; and (iii) the within contribution is Technology + Capital Intensity.

VI. Conclusions

The exact aggregation and decomposition methods introduced in this paper are uniquely suited to (i) compare the contribution of different groups of firms to aggregate productivity, (ii) gauge the importance of underlying micro patterns, and (iii) assess the importance of firm-level total factor productivity growth for aggregate output growth. The results for the Chinese manufacturing sector over the 2000-2005 period illustrate the type of insights we can obtain.

First, we learned that productivity differences between exporters and non-exporters and between firms of different ownership type have declined noticeably over the period studied. Even if it were possible to raise the productivity level of public

firms to that of domestic private firms, by 2005 this would make only a minor contribution to aggregate output.

Second, the productivity level decompositions illustrated that the negative association between size and productivity, which depresses aggregate productivity, has weakened over time. Moreover, the positive association between total factor productivity and capital intensity has grown stronger over time. Both patterns are encouraging for the future evolution of aggregate productivity.

Third, while the importance of net entry in a restructuring economy like China is expected, total factor productivity growth by incumbent firms is also a remarkably important determinant of aggregate output growth. In each case we studied, its contribution equals more than one half of the output growth caused by net entry; for foreign firms it even reached 84% of the net entry contribution. Capital deepening, on the other hand, is on average four times less important than productivity growth and the between firm effect is almost an order of magnitude smaller (or even negative).

APPENDIX - PROOFS

Proof of Proposition 1: Order firms by increasing labor productivity such that $LP_1 \leq LP_2 \dots \leq LP_N$. With some algebraic manipulations we can sign the following expression,

$$\begin{aligned}
 & \sum_i \theta_i^Q LP_i - \sum_i \theta_i^L LP_i \\
 &= \sum_i (\theta_i^Q - \theta_i^L) LP_i \\
 &= \sum_{i>1} (\theta_i^Q - \theta_i^L) (LP_i - LP_1) \\
 &\geq \sum_{i>1} (\theta_i^Q - \theta_i^L) (LP_2 - LP_1) \text{ because } LP_1 \leq LP_2 \leq LP_i \quad \forall i > 2 \\
 &= [(1 - \theta_1^Q) - (1 - \theta_1^L)] (LP_2 - LP_1) \\
 &= \left[\frac{L_1}{\sum_i L_i} = \frac{Q_1}{\sum_i Q_i} \right] (LP_2 - LP_1)
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{L_1}{\sum_i Q_i} \left[\frac{\sum_j Q_j}{\sum_j L_j} - \frac{Q_1}{L_1} \right] (LP_2 - LP_1) \\
 &= \frac{L_1}{\underbrace{\sum_j Q_j}_+} \left[\underbrace{LP}_+ - \underbrace{LP_1}_+ \right] (LP_2 - LP_1) \\
 &\geq 0.
 \end{aligned}$$

Proof of Proposition 2: We can write the summation in the following form,

$$\sum_i \left(\frac{L_i}{\sum_i L_i} \right)^\alpha \left(\frac{K_i}{\sum_i K_i} \right)^{1-\alpha} \Rightarrow \sum_i L_i^\alpha K_i^{1-\alpha} \leq \left(\sum_i L_i \right)^\alpha \left(\sum_i K_i \right)^{1-\alpha}$$

With the following definitions,

$$p = 1 / \alpha \quad q = 1 / (1 - \alpha)$$

$$a_i = L_i^\alpha \quad b_i = K_i^{1-\alpha},$$

the inequality then follows directly from *Holder's inequality*:

$$\sum_i |a_i b_i| \leq \left(\sum_i |a_i|^p \right)^{\frac{1}{p}} \left(\sum_i |b_i|^q \right)^{\frac{1}{q}}.$$

NOTES

1. Department of Economics, University of Toronto, 150 St. George Street, Toronto ON M5S 3G7, Canada, Tel. +1 416 946 5795, E-mail: jobv@chass.utoronto.ca. I would like to thank Yifan Zhang, Loren Brandt, and Luhang Wang for help with the data; Geert Dhaene for help with some derivations; and an anonymous referee for comments. Financial support from the Connaught Fellowship is gratefully acknowledged.
2. Foster, Haltiwanger, and Krizan (2001) provide an overview of the decomposition literature.
3. When referring to the literature I will often refer to my own work (available from my web site) if it is representative. More extensive references to other researchers' work can be found in those papers.
4. In the empirical section of the paper, we work with a sample of Chinese manufacturing firms. Estimating returns to scale freely by sector gives an average of 0.93, slightly lower in 2000-2002 and slightly higher in 2003-2005. Results in Van Biesebroeck (2007) indicate that the exact assumption on the returns to scale is unlikely to materially affect the results.

5. In the theoretical derivations, we mostly use firms to indicate the unit of observation, but all derivations hold equally well at the plant level. In the empirical illustration, we work at the firm level.
6. These formulas are slightly different under a counterfactual where only a fraction of the firms have an increased level of total factor productivity, but differences among firms remain.
7. With a more sophisticated production function, employing more inputs or accounting for more decision variables, such a decomposition can be pushed further – see Van Biesebroeck (2003) for an application that introduces intermediates, scale economies, and the explicit choice of the number of production shifts.
8. Note, however, that for our choice of input weighing and firm weights this will only hold approximately. This issue is discussed in greater detail below and in Petrin and Levinsohn (2006).
9. Total factor productivity growth as defined here will differ from the often-used Solow growth decomposition, because the weight we use on labor input does not vary over time. Throughout, we use a fixed α rather than the average labor share $(S_t + S_{t-1})/2$.
10. In general, we can write for arbitrary k

$$(S) = \sum_i \left((k\theta_{it-1} + (1-k)\theta_{it}) \frac{\Delta P_{it}}{P_{t-1}} + \Delta\theta_{it} \frac{(1-k)P_{it-1} + kP_{it}}{P_{t-1}} \right),$$

where the value of $k \in (0,1)$ will influence the relative importance of both terms. Choosing $k = 1/2$ is intuitive, but only one of many possibilities.

11. Gross output equals the sum of value added (Q_t) and materials and intermediate inputs (M_t).
12. He provides two alternatives how to define aggregate growth as a quantity index, with appropriate decompositions.
13. Dividing in the first equation by \bar{Q} would be more intuitive, but it would not be exact as $\bar{Q} = LPL + \Delta LPA L / 4$. Moreover, the division of the last equation by \bar{LP} requires the pre-multiplication of both right-hand side terms by $\frac{TFP(K/L)^{1-\alpha}}{LP}$, if one wants to normalize the TFP factors by $\frac{TFP}{LP}$ and the K/L factors by $\left(\frac{K}{L}\right)^{1-\alpha}$. The ratio differs slightly from one as $\bar{LP} = \frac{TFP(K/L)^{1-\alpha}}{LP} + \Delta TFP_t \Delta(K_t/L_t)^{1-\alpha} / 4$.
14. We define public firms broadly, including state-owned, collective, and cooperative enterprises.
15. The hypothetical output increases are calculated using the formulas for ΔQ_t from Section 2.
16. While the two subsectors are not mutually exclusive, the overlap is quite small.

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